

**Dominik Wachla, Andrzej Katunin\***

*Silesian University of Technology, Institute of Fundamentals of Machinery Design, ul. S. Konarskiego 18A, 44-100 Gliwice, Poland*

*\* Corresponding author. E-mail: andrzej.katunin@polsl.pl*

*Received (Otrzymano) 18.10.2018*

## MODELLING FATIGUE WITH SELF-HEATING OF POLYMERIC COMPOSITES BASED ON STATISTICAL ANALYSIS OF TEMPERATURE PROFILES

The self-heating effect occurring during the fatigue loading of polymeric composite structures subjected to cyclic loading or vibration is a result of energy dissipation appearing due to the viscoelastic properties of the matrix of such composites. The occurrence of the self-heating effect during the operation of structural elements is very dangerous, since the increasing self-heating temperature intensifies the initiation and propagation of fatigue damage, and may significantly shorten the residual life of such elements. Following this, it is necessary to control this process. The theoretical models developed to date may be inaccurate in predicting the residual life of a structure subjected to fatigue with the appearance of self-heating, especially after the initiation of structural damage. Therefore, the authors proposed the empirical model based on estimating the parameters from the self-heating temperature profiles with statistical analysis of these parameters, which allows one to determine the estimators and predict the residual life of composite structures working in such conditions in more accurate way.

**Keywords:** fatigue, polymeric composites, self-heating effect, modelling damage evolution

## MODELOWANIE ZMĘCZENIA Z SAMOROZGRZANIEM KOMPOZYTÓW POLIMEROWYCH NA PODSTAWIE ANALIZY STATYSTYCZNEJ PROFILI TEMPERATUROWYCH

Effekt samorozgrzania, powstający podczas obciążeń zmęczeniowych struktur wykonanych z kompozytów polimerowych poddanych cyklicznemu obciążaniu lub drganiom, jest wynikiem dyssypacji energii, inicjowanej ze względu na lepkość sprężyste właściwości osnowy tych kompozytów. Powstawanie efektu samorozgrzania podczas eksploatacji elementów strukturalnych jest bardzo niebezpieczne, gdyż wzrost temperatury samorozgrzania intensyfikuje inicjację i propagację uszkodzeń zmęczeniowych i może znacząco obniżyć trwałość resztkową tych elementów. Mając to na uwadze, należy kontrolować ten proces. Dotychczas opracowane modele teoretyczne mogą być niedokładne w predykcji trwałości resztkowej struktury poddanej zmęczeniu z występującym samorozgrzaniem, zwłaszcza po inicjacji uszkodzeń strukturalnych. Dlatego autorzy zaproponowali model empiryczny oparty na estymacji parametrów na podstawie profili temperaturowych samorozgrzania z analizą statystyczną tych parametrów, co pozwala wyznaczyć estymatory i przewidywać trwałość resztkową struktur kompozytowych pracujących w takich warunkach z większą dokładnością.

**Słowa kluczowe:** zmęczenie, kompozyty polimerowe, efekt samorozgrzania, modelowanie przyrostu uszkodzeń

### INTRODUCTION

When polymeric composites have been operated under loads with a high amplitude, a significant increase in temperature can be observed in locations with stress concentrations. The temperature increase in these locations is caused mainly by the thermoviscoelastic properties and poor thermal conductivity of most industrial polymers used for manufacturing composite structures. The phenomenon of temperature increase in polymeric composites is commonly known as the self-heating effect [1]. The results of previously performed research studies concerning the fatigue strength of polymeric composites revealed that the self-heating effect causes a significant decrease in the fatigue limit of those structures, and it depends on factors such as load amplitude

and its frequency [1-3]. Depending on the scenario of the self-heating evolution and the amount of generated energy, the structural lifetime of an operated element in such conditions may be considerably shortened [4].

The influence of the fatigue of polymeric composite structures with the accompanying self-heating effect has been deeply studied since the 1960s when Ratner and Korobov [5] and Oldyrev [6] started studying the structural degradation of polymers subjected to cyclic loading. Due to the mechanical origin of the self-heating effect, the results of their works were extended to polymeric composites, and the first fatigue models that consider the self-heating effect appeared during that period [7-9]. These models were based on

Zhurkov's kinetic concept of structural degradation. The fatigue model in similar conditions was developed independently by the authors of [10], which was based on incremental crack propagation criterion.

Nowadays, the problem of the fatigue of polymeric composites accompanied by the self-heating effect still needs additional investigation and is still attractive for numerous researchers. A growth of interest in these phenomena has been observed in the last two decades. In particular, the authors of [11] investigated the self-heating induced fatigue process for woven composite laminates under cyclic tension, and formulated a fatigue model based on damage accumulation and a decrease in structural stiffness. The authors of [12] approximate the results of fatigue tests of a polyamide 66/glass composite under cyclic bending obtained in the form of S-N curves and proposed a simple logarithmic fatigue model. A combination of analytical modelling and experimental testing of polyamide 6 specimens under tension loading proposed by the authors of [13] allowed them to obtain a hybrid fatigue model that considers self-heating. The studies of Tamuzs et al. [14] deserve special attention, where the authors performed fatigue tests of glass fibre/vinyl ester woven composites, and based on the obtained results developed the polynomial fatigue model. The previous studies of the second author of the present paper have focused both on theoretical modelling and experimental studies. In particular, the model presented in [15] was based on a decrease in rigidity dependent on the temperature and excitation frequency, and simultaneously on the damage growth concept. Then the model was extended using the dynamic properties of a composite, which allowed the author to consider the thermal component of the fatigue process [16]. In a similar way, i.e. using the fundamentals of continuum mechanics of thermoviscoelasticity, the authors of the very recent studies of fatigue with self-heating developed prediction fatigue models [17, 18].

In contrast to the above described models, one can consider structural degradation from the point of view of characteristic phenomena occurring during the whole degradation process using the results of numerous measurement studies. This allows modelling of structural degradation during fatigue using a set of parameters describing the model. Years of experiments allow to one perform of statistical analysis of the mentioned parameters and describe the model in terms of the probability of degradation during non-stationary self-heating, which is the main goal of this study.

## SELF-HEATING EFFECT AND APPROXIMATION MODEL

The self-heating temperature evolution is well described by the three-phase model [1, 2]. The resulting temperature profile consists of three phases: initial increase (phase I), steady-state evolution (phase II), and

abrupt growth to final failure (phase III), for instance see Figure 1.

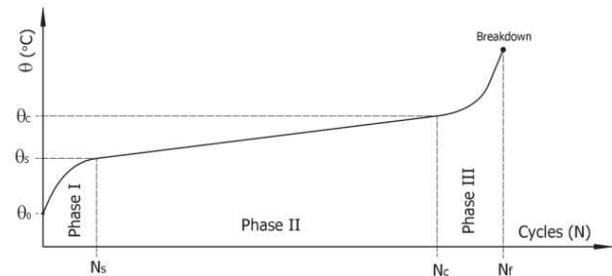


Fig. 1. Typical temperature profile during self-heating of polymeric composites

Rys. 1. Typowy profil temperaturowy podczas samorozgrzania kompozytów polimerowych

Each of the phases is limited to a specific temperature  $\theta$  and number of load cycles  $N$ . In fact, only temperature  $\theta_0$  of the ambient medium and the  $\theta_f$  and  $N_f$  are known from the fatigue test. The others, i.e.  $\theta_s$ ,  $\theta_c$ ,  $N_s$ , and  $N_c$ , are unknown in general, but they can be determined using, for example, a double-exponential approximation model [1]:

$$\theta(N) = \Gamma_1 \exp(\lambda_1 N) + \Gamma_2 \exp(\lambda_2 N), \quad (1)$$

where:  $N$  - number of load cycles,  $\Gamma_1$  - specimen temperature at  $N_s$ ,  $|\Gamma_2|$  - increase in specimen temperature at  $N_s$ ,  $\lambda_1$  and  $\lambda_2$  - specimen heating rate in the first and the second phases, respectively. Based on the parameters in (1), features  $\theta_0$  and  $\theta_s$  (Fig. 1) can be identified as follows:  $\hat{\theta}_0 \equiv \Gamma_1$  and  $\hat{\theta}_s \equiv \Gamma_1 + |\Gamma_2|$ . Other features, i.e.  $\theta_c$ ,  $N_s$ , and  $N_c$ , can be found by means of comparing the experimental fatigue temperature profile with its approximation following (1).

Previous studies [1, 2] show that features describing the fatigue temperature profile of polymeric composites, i.e.  $\theta_s$ ,  $\theta_c$ ,  $N_s$ ,  $N_c$ ,  $\lambda_1$ , and  $\lambda_2$ , depend on the material properties, loading amplitude and its frequency. On the other hand, the fatigue tests of polymeric composites are conducted in different environmental conditions e.g. at different ambient temperature  $\theta_0$ . In many previous investigations concerning the fatigue strength of polymeric composites (see e.g. [1-3, 11, 12, 14]), the influence of ambient temperature  $\theta_0$  on the fatigue limit was not considered. However, observations of the self-heating temperature evolution during fatigue tests conducted in different environmental conditions proves that  $\theta_0$  is an important factor and it determines the final results of fatigue tests i.e. the values of features used to describe the fatigue temperature profile (Fig. 1). For this reason, statistical dependency analysis between  $\theta_0$  and the remaining features describing the fatigue temperature profile need to be performed. Additionally, statistical tests, correlation, regression, and other analyses are necessary to identify the dependency between  $\theta_0$  and other considered features. The results of these

analyses are presented in the next section, and they will allow description of the fatigue processes as well as modelling of the self-heating effect in terms of empirical features obtained from the experimental data.

### ANALYSIS OF TEMPERATURE PROFILES

#### Acquisition of temperature profiles

Temperature profiles were acquired during fatigue testing of glass fibre-reinforced polymeric composite specimens with an effective length of 50 mm, width of 10 mm, and thickness of 2.5 mm. Details on the manufacturing process as well as the mechanical and thermal properties of the specimens can be found in [16]. All the analyses were conducted using selected fatigue temperature profiles which were obtained from 4 different experiments on the experimental test rig described in [16] with varying thermal conditions and the same loading conditions, i.e. loading force of 90 N, and loading frequency of 30 Hz, as well. The thermal response of the surfaces of the specimens subjected to such loading was registered by an infrared camera. Then from the sequence of thermograms, temperature profiles of the type presented in Figure 1 were extracted and subjected to further analysis. In this study 84 temperature profiles were considered for the analysis.

#### Analysis of basic statistical features

From the set of temperature profiles the statistical features were calculated in order to perform initial data analysis, especially the sensitivity of particular features to changes was analyzed. In this study the following features were considered: minimum and maximum val-

ues, range, mean and median values, standard deviation as well as higher-order moments: kurtosis and skewness. The results of the performed study are presented in Table 1. The results of the statistical analysis do not reveal any tendencies, therefore deep study of the analyzed data using more advanced statistical tools is necessary.

TABLE 1. Results of basic statistical analysis of self-heating characteristic times and temperatures

TABELA 1. Wyniki podstawowej analizy statystycznej charakterystycznych czasów i temperatur samorozgrzania

	$\theta_0$ [°C]	$t_s$ [s]	$\theta_s$ [°C]	$t_c$ [s]	$\theta_c$ [°C]
Min.	18.32	22.0	36.16	135.5	50.05
Max.	31.30	395.0	65.67	1141.0	83.19
Range	12.97	373.0	29.51	1005.5	33.14
Mean	24.09	111.2	48.90	403.2	60.94
Median	23.43	101.3	50.12	356.0	61.00
Standard deviation	2.46	50.5	5.30	165.1	6.91
Kurtosis	3.13	13.2	0.77	4.6	2.31
Skewness	1.71	3.0	0.14	1.7	1.15

#### Correlation analysis and statistical tests

In order to identify the relations between the considered features, a matrix of scatter plots was prepared and the values of three correlation coefficients, i.e.  $r$  - the Pearson correlation coefficient [19],  $\rho$  - Spearman's rank correlation coefficient [19] and  $\tau$  - the Kendall rank correlation coefficient [19, 20] (Fig. 2) were calculated.

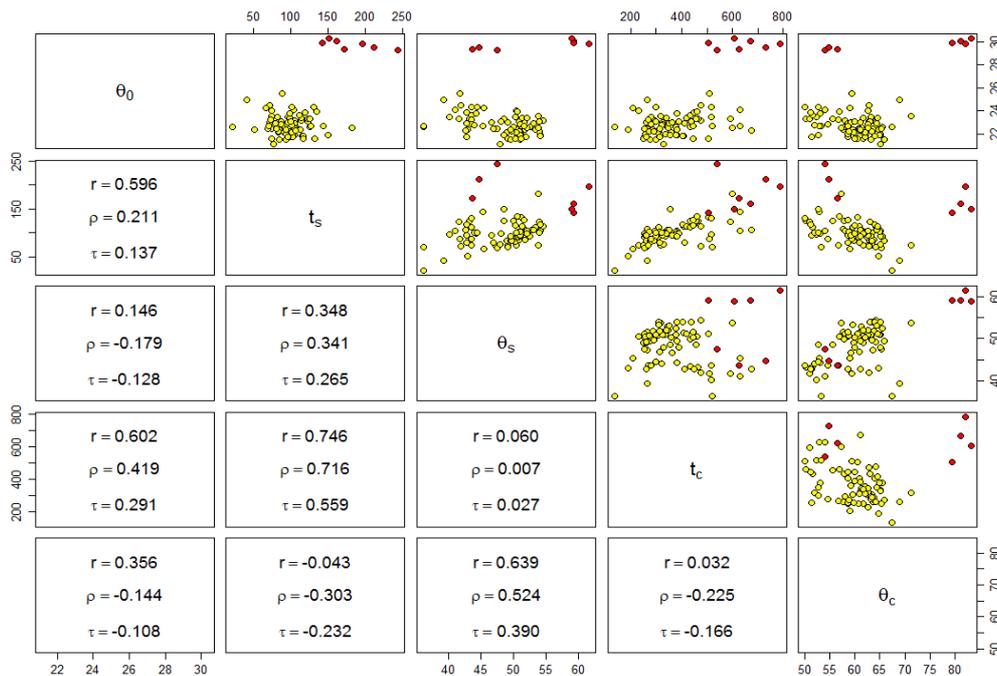


Fig. 2. Scatter plot matrix and correlation analysis of features describing self-heating effect

Rys. 2. Macierz wykresów rozproszenia oraz analiza korelacyjna cech opisujące efekt samorozgrzania

The correlation analysis results (Fig. 2) revealed that there are at least three interesting associations in the considered data set. Firstly, in the case of comparison of  $\theta_0$  with other features, one can observe that the full data set can be divided into two groups. The first is focused around the  $\theta_0$  temperature of ca. 23°C (yellow dots in Fig. 2), and the second one around the  $\theta_0$  temperature of ca. 30°C (red dots in Fig. 2). This leads to the second observation where on the basis of scatter plots between  $\theta_0$  and  $t_s$ , and  $\theta_0$  and  $t_c$  (Fig. 2), the impact of the test/ambient temperature (identical to the initial specimen temperature) of the polymer composite on its fatigue durability, i.e. strength in the considered range of 20÷30°C of ambient temperature  $\theta_0$  can be inferred. The third observation concerns the occurrence of a relatively high correlation between  $t_s$  and  $t_c$  characteristics, which indicates that these parameters are linearly dependent, despite the typically heuristic approach when determining the value of the  $t_c$  parameter using the approximate model (1).

The first and second observations allowed the authors to formulate a hypothesis concerning the influence of the initial temperature of a polymeric composite specimen (identical to ambient temperature) on its fatigue strength. In order to prove this hypothesis the collected data (values) were divided into two samples in the first round: the first sample with the designation *LTemp* with data for which the  $\theta_0$  feature values were close to 23°C, and the second sample with the designation *HTemp* with data for which the  $\theta_0$  feature values were close to 30°C. The basic statistics were determined for both samples (see Table 2).

Formal verification of the posed hypothesis can be performed, among others, using a two-sample *t*-test for mean equality [19, 21]. This test, however, requires, among others, independence of both the analyzed samples and their population should be characterized by normal distribution. In the case of the considered data the independence requirement was fulfilled, which

results from the specification of fatigue testing of polymeric composites, where each specimen, due to the destructive character of testing, is a source of only one set of data. In the case of the second of the above-mentioned requirements, checking the compliance of the value distribution of particular features determined from the measurement data with a normal distribution is necessary. This results from the fact that there are no known research studies performed to-date.

For this purpose several statistic tests were developed. One of these tests is the Shapiro-Wilk test [22]. Within this test, the null hypothesis is assumed:  $H_0$ : "The data came from a normally distributed population" and the alternative hypothesis  $H_1$ : "The data are not normally distributed". In order to verify both hypotheses, the value of statistic  $W$  of the Shapiro-Wilk distribution is calculated [22] and compared further with critical value  $W_{crit}$  obtained from the tables of the Shapiro-Wilk distribution [22] for the assumed significance level  $\alpha$  and the size of the test sample. In the case when the computational value of a statistic  $W$  is bigger than the table value, there is no reason to reject the null hypothesis, otherwise the null hypothesis should be rejected and the alternative hypothesis accepted.

Table 3 presents the obtained results of verifying the compliance of the value distribution of the considered features with normal distribution for both the *LTemp* and *HTemp* data samples. In turn, Figure 3 shows the histograms of the empirical distribution of the considered characteristics for an *LTemp* data sample together with the theoretical normal distribution curve calculated on the basis of the estimated mean value and standard deviation (see Table 2). The results from Table 3 (and analysis of the graphs in Figure 3 in the case of the *LTemp* sample) show that the value distributions of features  $\theta_0$  and  $t_s$  have a normal distribution. One can notice that the  $t_s$  feature may be considered to be a feature with value distribution described by normal distribution, as well.

TABLE 2. Basic statistics of selected features from specimens (groups) *LTemp* and *HTemp*  
TABELA 2. Podstawowe statystyki dla wybranych cech dla próbek (grup) *LTemp* oraz *HTemp*

	Sample ID: <i>LTemp</i> , Sample size: 75					Sample ID: <i>HTemp</i> , Sample size: 7				
	$\theta_0$ [°C]	$t_s$ [s]	$\theta_s$ [°C]	$t_c$ [s]	$\theta_c$ [°C]	$\theta_0$ [°C]	$t_s$ [s]	$\theta_s$ [°C]	$t_c$ [s]	$\theta_c$ [°C]
Min.	21.1	22	36.2	135	50.1	29.3	143	43.7	507	53.9
Max.	25.5	182	54.5	673	71.2	30.3	243	61.6	787	83.2
Mean	22.8	98.3	48.2	364.5	59.9	29.8	182.3	53.5	638.4	70.2
Median	22.7	99	50	339.5	61	29.8	171.5	58.9	625.5	79.5
St. dev.	0.873	24.065	4.503	110.063	4.826	0.386	36.337	7.861	100.016	14.192
Variance	0.76	579.14	20.28	12113.84	23.29	0.149	1320.405	61.803	10003.2	201.403
Kurtosis	3.317	4.997	2.7	3.234	2.505	1.676	2.063	1.227	1.871	1.113
Skewness	0.671	0.115	-0.762	0.753	-0.402	0.179	0.561	-0.308	0.148	-0.279

TABLE 3. Shapiro-Wilk normality test results for selected features of *LTemp* and *HTemp* samples  
 TABELA 3. Wyniki testu normalności Shapiro-Wilka dla wybranych cech dla próbek *LTemp* oraz *HTemp*

Feature ID	Sample ID: <i>LTemp</i> , Sample size: 75		Sample ID: <i>HTemp</i> , Sample size: 7	
	Assumed significance level $\alpha = 0.05$		Assumed significance level $\alpha = 0.05$	
	Critical value of $W_{0.05} = 0.967$		Critical value of $W_{0.05} = 0.803$	
	Test statistic W	<i>p</i> -Value	Test statistic W	<i>p</i> -Value
$\theta_0$ [°C]	<b>0.96896</b>	<b>0.06226</b>	<b>0.94655</b>	<b>0.6982</b>
$t_s$ [s]	<b>0.96863</b>	<b>0.05953</b>	<b>0.93803</b>	<b>0.6210</b>
$\theta_s$ [°C]	0.91386	8.354e-05	0.80049	0.0414
$t_c$ [s]	0.94851	0.004136	<b>0.97343</b>	<b>0.9220</b>
$\theta_c$ [°C]	0.95346	0.007750	0.74506	0.0113

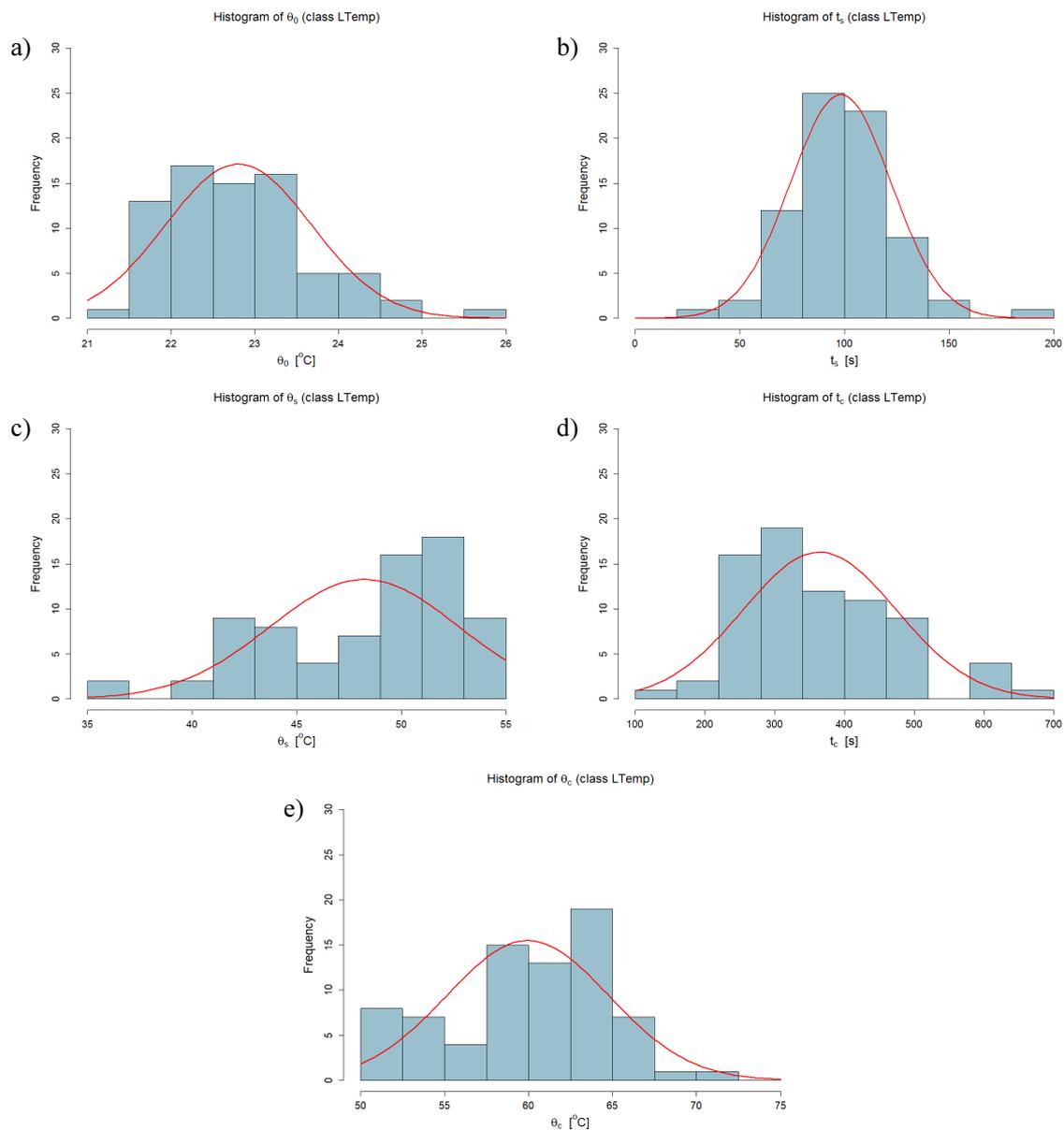


Fig. 3. Histograms of selected features of *LTemp* and *HTemp* samples for: a)  $\theta_0$ , b)  $t_s$ , c)  $\theta_s$ , d)  $t_c$ , and e)  $\theta_c$

Rys. 3. Histogramy dla wybranych cech dla próbek *LTemp* oraz *HTemp* dla: a)  $\theta_0$ , b)  $t_s$ , c)  $\theta_s$ , d)  $t_c$ , e)  $\theta_c$

Since the size of the *HTemp* sample is small, which results in a decrease in the reliability (power) of the Shapiro-Wilk test [22], additional analyses were performed to

assess the compatibility of the value distribution of features  $\theta_0$  and  $t_s$  with normal distribution. The analyses were based on a quantile plot for normal distribution (Fig. 4).

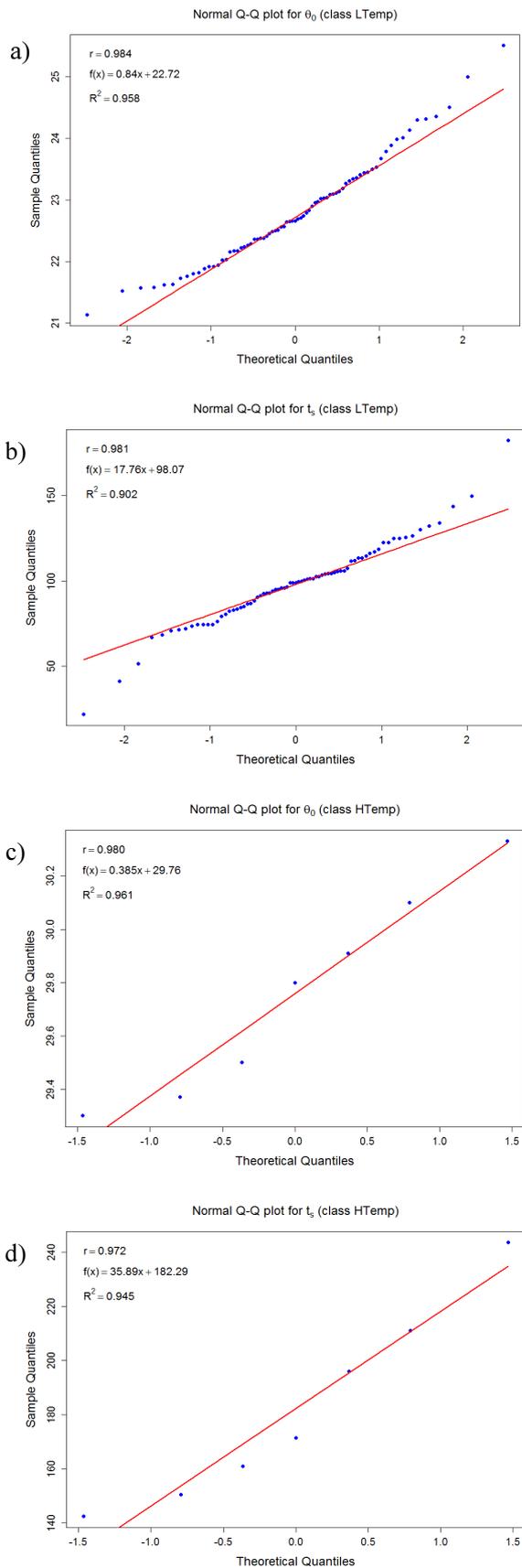


Fig. 4. Qualitative analysis for normal distributions of features  $\theta_0$  and  $t_s$  from *LTemp* and *HTemp* samples using quantile-quantile plots

Rys. 4. Analiza ilościowa dla rozkładów normalnych dla cech  $\theta_0$  i  $t_s$  dla próbek *LTemp* oraz *HTemp* na podstawie wykresów kwantylowych

In order to conduct quantile-quantile plot analysis, the theoretical quantiles  $z_i$  were calculated using following formula [23]:

$$z_i = \Phi^{-1}\left(\frac{i-0.5}{n}\right) \quad (2)$$

where:  $\Phi$  - the standard normal quantile function [19, 21],  $n$  - size of sample,  $i$  - quantile index.

Next, sample quantiles  $y_i$  were sorted in ascending order and then associated with appropriate theoretical quantiles (2) forming a set of points  $(z_i, y_i)$  for the quantile-quantile (Q-Q) plot in Figure 4.

In general, the analysis of Q-Q plots is simple and is based on observing whether the Q-Q points lie in a line. The occurrence of linearity between the theoretical quantiles of a standardized normal distribution and empirical quantiles indicates the origin of a sample of data from a population with a normal distribution.

Formal verification of the significance of the occurrence of linear dependence between the theoretical and empirical quantiles can be carried out using the test of significance for a Pearson product-moment correlation coefficient  $r_p$  [19, 21]. Use of the above test requires the source data to come from a normal distribution population and that their scale is the interval scale. The test statistic has a Student's  $t$ -distribution with  $n - 2$  degrees of freedom and is given in the form of [19]:

$$t = r_p \sqrt{\frac{n-2}{1-r_p^2}} \quad (3)$$

Within the test, hypothesis zero  $H_0: r_p = 0$  is assumed for the lack of correlation between variables in the population and the alternative hypothesis  $H_1: r_p \neq 0$  assuming a significant correlation between the variables. The critical value of the  $t$ -statistic is read from the Student's  $t$ -distribution tables for the assumed significance level  $\alpha$  and the number of degrees of freedom equal to  $n - 2$ , where  $n$  is the sample size. The obtained results of testing the correlation coefficient for data shown in the quantile-quantile plots are summarized in Table 4. Additionally, on individual graphs (Fig. 4) an equation of linear regression is presented with the value of the  $R^2$  coefficient determining the quality (goodness) of matching.

The quantile-quantile plot analyses (Fig. 4) of the normal distribution presented above, and the results of the correlation significance test (Table 4) between the theoretical quantiles of a standardized distribution and empirical quantiles confirmed the results (Table 3) obtained using the Shapiro-Wilk test [22]. Confirmation of the normality of the  $\theta_0$  and  $t_s$  feature distributions in the *LTemp* and *HTemp* populations allowed the authors to move to the essential stage of verifying the initial hypothetical separation of the impact of initial temperature  $\theta_0$  of the polymeric composite sample on its fatigue strength, which is indirectly represented by the  $t_s$  parameter. It was assumed that this verification will take

place by comparing the mean values for  $\theta_0$  and  $t_s$  in the *LTemp* and *HTemp* samples with the Student's *t*-test [19, 21]. Selecting a specific calculation procedure to determine the statistics *t* requires examining whether there is equality of variance of the considered feature in the two compared samples. Equity analysis of the variance is used among others by the F-test (Fisher), whose statistics are given by the formula [21]:

$$F = \frac{s_x^2}{s_y^2}, \tag{4}$$

where  $s_x^2$  and  $s_y^2$  are the variance values from the *x* and *y* samples. The *F*-statistic has the Snedecor distribution with  $\nu_1 = n - 1$  and  $\nu_2 = m - 1$  degrees of freedom if null hypothesis  $H_0: s_x^2 = s_y^2$  is satisfied. Hypothesis  $H_1: s_x^2 \neq s_y^2$  (two-tailed test) was assumed as an alternative hypothesis in the case of which the first critical value,  $F_{crit1} = F(\alpha/2, \nu_1, \nu_2)$  and the second one  $F_{crit2} = 1/F_{crit1}$ . The results of the comparison of the variance values of the two characteristics  $\theta_0$  and  $t_s$  between the *LTemp* and *HTemp* samples are presented in Table 5. The obtained results show that the variance of  $\theta_0$  and  $t_s$  in the *LTemp* and *HTemp* samples is significantly different. In turn, in the case of  $t_s$ , there is no basis for rejecting the null hypothesis.

TABLE 4. Results of testing of correlation significance between theoretical quantiles from standardized normal distribution and empirical quantiles of features from samples *LTemp* and *HTemp* (see Fig. 4)

TABELA 4. Wyniki badania istotności korelacji pomiędzy kwantylami teoretycznymi ze standaryzowanego rozkładu normalnego i kwantylami empirycznymi cech dla próbek *LTemp* oraz *HTemp* (patrz rys. 4)

Feature ID	Sample ID: <i>LTemp</i> , Sample size: 75			Sample ID: <i>HTemp</i> , Sample size: 7		
	Degrees of freedom: 73			Degrees of freedom: 5		
	Assumed significance level $\alpha = 0.05$			Assumed significance level $\alpha = 0.05$		
	Critical value of $t_{73} = 1.666$			Critical value of $t_5 = 2.015$		
	$r_p$	Test statistic <i>t</i>	<i>p</i> -Value	$r_p$	Test statistic <i>t</i>	<i>p</i> -Value
$\theta_0, ^\circ\text{C}$	0.984	47.682	< 2.2e-16	0.980	11.054	0.0001055
$t_s, \text{s}$	0.981	42.798	< 2.2e-16	0.972	9.302	0.0002416

TABLE 5. Equality of variance test results of two samples *LTemp* and *HTemp* for features  $\theta_0$  and  $t_s$

TABELA 5. Wyniki testu na równość wariancji w próbkach *LTemp* oraz *HTemp* dla cech  $\theta_0$  i  $t_s$

Feature ID	$\theta_0$ [ $^\circ\text{C}$ ]	$t_s$ [s]
Assumed significance level $\alpha$	0.05	0.05
Numerator degrees of freedom $\nu_1$	74	74
Denominator degrees of freedom $\nu_2$	6	6
Critical value of $F_{0.025}(\nu_1, \nu_2)$	4.9462	4.9462
Critical value of $F_{0.975}(\nu_1, \nu_2)$	0.2022	0.2022
Test statistic <i>F</i>	5.1045	<b>0.4386</b>
<i>p</i> -Value	0.0456	<b>0.0898</b>

Considering the results presented in Table 5, the Welch [24] approach for testing a two sample mean equality was applied in the case of feature  $\theta_0$  and the classic Student's *t*-test [19, 21] was applied in the case of feature  $t_s$ . In both cases null hypothesis  $H_0$  states that the mean values of two samples are equal  $\mu_x = \mu_y$ , while alternative hypothesis  $H_1$  assumes that the mean values are not equal  $\mu_x \neq \mu_y$ . For our problem, the above hypotheses were particularly defined as follows:

1.  $H_0$ : mean values of feature  $\{\theta_0, t_s\}$  in samples *LTemp* and *HTemp* are equal,
2.  $H_1$ : mean values of feature  $\{\theta_0, t_s\}$  in samples *LTemp* and *HTemp* are not equal.

To test the null hypothesis, test statistic *t* and degrees of freedom  $\nu$  were calculated using adequate formulas [19, 21, 24]. The critical values  $t_{0.025}$  and  $t_{0.975}$  of statistic *t* for a two-tailed test and significance level  $\alpha = 0.05$  were found using the tables of values from Student's *t*-distribution. The results of testing the mean equality of samples *LTemp* and *HTemp* for features  $\theta_0$  and  $t_s$  are presented in Table 6.

TABLE 6. Results of testing of mean equality of two samples *LTemp* and *HTemp* for features  $\theta_0$  and  $t_s$

TABELA 6. Wyniki testu na równość wartości średniej w próbkach *LTemp* oraz *HTemp* dla cech  $\theta_0$  i  $t_s$

Feature ID	$\theta_0$ [ $^\circ\text{C}$ ]	$t_s$ [s]
Assumed significance level $\alpha$	0.05	0.05
Degrees of freedom $\nu$	13	80
Critical value for $t_{0.025}$	-2.160	-1.990
Critical value for $t_{0.975}$	2.160	1.990
Test statistic <i>t</i>	39.184	9.302
<i>p</i> -Value	9.482e-15	1.14e-12

The results presented in Table 6 show that difference of the mean value of  $\theta_0$  and  $t_s$  in the compared samples *LTemp* and *HTemp* is significant. They prove the correctness of the assumed hypothesis about the influence of the initial temperature of a polymeric composite sample on its durability.

## DISCUSSION AND CONCLUSIONS

The obtained results of statistical analyses confirm the accepted hypothesis that a change in the initial temperature of the polymeric composite resulting from a change in the ambient temperature in the range from 20 to 30 $^\circ\text{C}$ , leads to a change in its durability. This change has a progressive character (growth), which means that the fatigue strength of a polymeric composite increases with an increase in its initial temperature identical to the ambient temperature. With regard to the self-heating effect, the observed effect of strength increase may be counter-intuitive. Probably the occurrence of this phenomenon is related to changes in the

crosslinking structure of the polymer matrix of the composite caused by a change in the ambient temperature. This hypothesis, however, requires additional microscopic examination. Final confirmation of the obtained results will be possible after performing detailed fatigue tests in an air-conditioning chamber, which will allow controlled parameters of the ambient atmosphere, including temperature to be obtained.

### Acknowledgements

*The results presented in this paper were obtained within the framework of research grant No. 2015/17/D/ST8/01294 financed by the National Science Centre, Poland.*

### REFERENCES

- [1] Katunin A., Critical self-heating temperature during fatigue of polymeric composites under cyclic loading, *Composites Theory and Practice* 2012, 12(1), 72-76.
- [2] Katunin A., Fidali M., Fatigue and thermal failure of polymeric composites subjected to cyclic loading, *Advanced Composites Letters* 2012, 21(3), 63-69.
- [3] Montesano J., Fawaz Z., Bougherara H., Use of infrared thermography to investigate the fatigue behavior of a carbon fiber reinforced polymer composite, *Composite Structures* 2013, 97, 76-83.
- [4] Katunin A., Domination of self-heating effect during fatigue of polymeric composites, *Procedia Structural Integrity* 2017, 5, 93-98.
- [5] Ratner S.B., Korobov V.I., Self-heating of plastics during cyclic deformation, *Polymer Mechanics* 1965, 1(3), 63-68.
- [6] Oldyrev P.P., Self-heating and failure of plastics under cyclic loading, *Polymer Mechanics* 1967, 3(3), 322-328.
- [7] Oldyrev P.P., Tamuzs V.P., Energy dissipation in a glass-reinforced plastic during prolonged cyclic deformation, *Strength of Materials* 1969, 1(3), 244-248.
- [8] Oldyrev P.P., Malinskii A.M., Effect of the temperature on high-cycle fatigue of organic-fiber-reinforced plastic, *Mechanics of Composite Materials* 1983, 19(2), 192-197.
- [9] Oldyrev P.P., Allowance for the heating of a glass-fiber plastic in high cycle fatigue tests, *Mechanics of Composite Materials* 1987, 24(1), 39-43.
- [10] Constable I., Williams J.G., Burns D.J., Fatigue and cyclic thermal softening of thermoplastics, *Journal of Mechanical Engineering Science* 1970, 12(1), 20-29.
- [11] Toubal L., Karama M., Lorrain B., Damage evolution and infrared thermography in woven composite laminates under fatigue loading, *International Journal of Fatigue* 2006, 28(12), 1867-1872.
- [12] Bellenger V., Tcharkhtchi A., Castaing P., Thermal and mechanical fatigue of a PA66/glass fibers composite material, *International Journal of Fatigue* 2006, 28(10), 1348-1352.
- [13] Ramkumar A., Kannan K., Gnanamoorthy R., Experimental and theoretical investigation of a polymer subjected to cyclic loading conditions, *International Journal of Engineering Science* 2010, 48(2), 101-110.
- [14] Tamuzs V., Dzelzitis K., Reifsnider K., Prediction of the cyclic durability of woven composite laminates, *Composites Science and Technology* 2008, 68(13), 2717-2721.
- [15] Katunin A., The conception of the fatigue model for layered composites considering thermal effects, *Archives of Civil and Mechanical Engineering* 2011, 11(2), 333-343.
- [16] Katunin A., Thermal degradation of polymer laminates, *Specialist Monographic Series "Library of Maintenance Problems"* no. 2336, The Publishing House of the Institute for Sustainable Technologies - National Research Institute, Radom 2012.
- [17] Lahuerta F., Westphal T., Nijssen R.P.L., Self-heating forecasting for thick laminate specimens in fatigue, *Journal of Physics: Conference Series* 2014, 555, 012062.
- [18] Mortazavian S., Fatemi A., Mellott S.R., Khosrovaneh A., Effect of cycling frequency and self-heating on fatigue behavior of reinforced and unreinforced thermoplastic polymers, *Polymer Engineering and Science* 2015, 55(10), 2355-2367.
- [19] Yadolah D., *The Concise Encyclopedia of Statistics*, Springer-Verlag, Berlin Heidelberg, 2008.
- [20] Kendall M.A., New measure of rank correlation, *Biometrika* 1938, 30(1-2), 81-89.
- [21] Snedecor G.W., Cochran W.G., *Statistical Methods*, 8-th edition, Iowa State University Press, 1989.
- [22] Shapiro S.S., Wilk M.B., An analysis of variance test for normality, *Biometrika* 1965, 52(3-4), 591-611.
- [23] Hazen A., Storage to be provided in the impounding reservoirs for municipal water supply, *Transactions of the American Society of Civil Engineers* 1914, 77, 1547-1550.
- [24] Welch B.L., The generalization of "Student's" problem when several different population variances are involved, *Biometrika* 1947, 34(1-2), 28-35.