CRACKING ANALYSIS OF REINFORCED CONCRETE BEAM ACCORDING TO PROPOSED METHODOLOGY OF PARAMETER SELECTION OF HIGH STRENGTH CONCRETE

The article presents the analysis of failure for a reinforced concrete beam with a low level of reinforcement, made of high strength concrete, in the process of static deformation in comparison to experimental results. On the basis of its ultimate uniaxial compressive strength, the methodology for defining the parameters of the constitutive model for high strength concrete was developed. The comparison of the obtained results indicates the correctness of the assumptions and constitutive models of the high strength concrete and reinforcement steel, and also the effectiveness of Crisfield’s arc length method and the Newton-Raphson iterative solution with adaptive descent. The numerical results of the smeared crack patterns proved to be qualitatively agreeable, regarding the localisation, direction and concentration, with the experimental results.

Keywords: finite element method, reinforced concrete, beam, high strength concrete

INTRODUCTION

High strength concretes are new construction materials that are increasingly common in the construction industry. The material is characterised by its high strength, low absorbability, low water permeability and high freeze resistance, which results in high durability. They are used in high buildings, drilling platforms, bridges, underground and hydro-technical constructions. Its common use is hindered by a lack of normative regulations and recommendations for design calculations. Admittedly, an increasingly higher number of implementations of new materials in the world construction industry is based on engineers’ experience and previous experiments to a larger extent, rather than on theoretical analyses. This results mainly from the high complexity of these composites and the properties that distinguish them from conventional concrete.

Precise simulation of these composites’ behaviour under static load requires the consideration of physical non-linearities of the construction component materials: high strength concrete and reinforcing steel, particularly material softening (non-static material behaviour), interaction on the contact surface between the concrete and steel, and considering the local concentration of large deformations influencing concrete cracking and crushing. In order to solve such a complicated numerical task, it is necessary to construct a spatial model with high density meshing, to consider spatial tensile stresses and deformations, and to use an appropriate method of solving simultaneous equations.

METHODOLOGY OF SELECTING CONCRETE MODEL PARAMETERS

The article presents the analyses of behaviour of a model reinforced high strength concrete beam in the
process of static deformation. The Finite Element Method calculations were performed using the ANSYS program. The numeric model of a spatial reinforced high strength concrete beam uses the dimensions of a simply supported rectangular BP-1a beam, analysed and described by Kamińska in her work [1] - Figure 1.

The secant stiffness matrix is generated while solving linear and non-linear tasks concerning plasticity, stress stiffness of the construction with large strain and crushing concrete with consideration for the post-cracking stress relaxation.

The method is based on setting adaptive descent parameter $\xi$ during the equilibrium iterations. The secant stiffness matrix is generated when solving linear and non-linear tasks concerning plasticity, stress stiffness of the construction with large strain and crushing concrete with consideration for the post-cracking stress relaxation.

In this method, changeable load factor $\lambda$ is calculated in balance equations in the procedure of finite elements and has a value from the range $(-1,1)$.

The equation has the incremental form at substep $n$ and iteration $i$:

$$
\left[ K^T_i \right] \{ \Delta u_i \} - \Delta \lambda \left\{ F^i \right\} = \left\{ F_i^{nr} \right\}$$

where $\Delta \lambda$ - incremental load factor.

Mather [4] proved that it should be theoretically possible to express the criteria of concrete crushing in all possible combinations of stresses with one parameter - the uniaxial compressive strength of concrete. Generally, with the value of concrete strength under compression, it is possible to define a complete set of material.

$$
\left[ K^T \right] = \xi \left[ K^S \right] + (1 - \xi) \left[ K^T \right]
$$

where: $\left[ K^S \right]$ - secant stiffness matrix, $\left[ K^T \right]$ - tangent stiffness matrix, $\xi$ - adaptive descent parameter.
data, necessary to make calculations in the non-linear numerical model.

In the three presented numerical solutions of the BPla beam, all the parameters of high-strength concrete are calculated on the basis of one parameter - the experimentally defined uniaxial compressive strength. All the parameters of the reinforcing steel were adopted on the basis of the values adapted to construction safety measures and quoted in the norm for concrete construction design.

In the first version of the numerical solution, the model is described by a three parameter failure surface interpreted as a three-axis surface of high strength concrete in compression and tension, in accordance with the William-Warnke theory [5, 6] (Fig. 4), and by the author’s own proposal of the surface evolution in the function of its deformation (Fig. 5), which was elaborated on the basis of numerical analyses and experiments on reinforced concrete beams.

Figure 4 illustrates the three-axis failure surface divided into the hydrostatic component (describing volume changes) and the deviatoric component (describing shape changes). The longitudinal surface included in the axis of rotation \( \sigma_1 = \sigma_2 = \sigma_3 \) constitutes the hydrostatic cross-section (Fig. 4). The deviatoric cross-section lies on the normal to the axis of rotation.

The strain deviator is described by polar coordinates \( r \) and \( \theta \), where \( r \) is a radial vector locating the failure surface for any angle \( \theta \) from the range \( 0 \leq \theta \leq 60^\circ \). Angle \( \theta \), called Lode’s angle, is the angle between the projection of the main strain axis \( \sigma_1 \) onto the deviator surface with the direction of a fictitious deviator strain vector \( \bar{\sigma} \) on this plane. The failure surface is defined by the following equation:

\[
\frac{1}{f_c} \frac{\sigma_2}{z} + \frac{1}{f_c} \frac{\tau}{r(\theta)} = 1
\]

where: \( \sigma_2, \tau \) - average values of normal and tangent stresses, \( z \) - vertex of failure surface, \( f_c \) - uniaxial compressive strength.

The angles of linear slant heights of the hydrostatic cone are described by \( \phi_1 \) and \( \phi_2 \). The parameters of the failure surfaces, \( z \) and \( r \) depend on uniaxial compressive strength \( f_c \), uniaxial tension strength \( f_t \) and biaxial compressive strength \( f_{cb} \). The mathematical model of the failure surface of concrete is characterised by straight calculation of the model parameters on the basis of a standard strength test, convexes (without bends) and continuity of the concrete failure surface.

The essence of the law of evolution of the surface in the function of its deformation is to consider the elastic-plastic hardening phase and the material softening phase in the state of uniaxial compression as well as the experimentally confirmed, more extensive limit deformations in the construction members in combination with the deformations of the recorded control elements (Fig. 5).

\[ a) \]

\[ b) \]

In many models based on testing samples, the descending fragment of the curve is more inclined to higher strength concrete, which proves that high strength concrete is more fragile. The relation is not always reflected in the concrete behaviour in construction elements made of reinforced concrete. Moreover, the results of laboratory tests showed that the fears that high strength concrete is characterised by low plastic strain are unjustified. The observed advantages are
profitable for construction safety. Applying too low strain values for the compression of high strength concrete in the model, in accordance with e.g. Model Code 90 [7], results in a significant decrease in the failures curves of the construction elements. The construction properties of the BP-1a beam are characterised by the following parameters of constituting models:

**High strength concrete**
- uniaxial compressive strength, experimentally defined by Kamińska [1], $f_c = 81.2$ MPa
- modulus of elasticity, calculated according to ACI 363 norm $E_c = 3.32\sqrt{f_c} + 6.9$, $E_c = 36817$ MPa
- uniaxial tension strength, calculated according to ACI 363 norm, $f_t = 0.54\sqrt{f_c}$, $f_t = 4.87$ MPa
- Poisson’s ratio $\nu_c = 0.15$
- concrete density $\rho_c = 2600$ kg/m$^3$
- compressive strain in concrete at peak stress $\varepsilon_{c1} = 6\%$
- ultimate compressive strain in concrete $\varepsilon_{cu} = 12\%$
- shear transfer coefficients for open crack $\beta_o = 0.5$, estimated on basis of author’s numerical analyses
- shear transfer coefficients for closed crack $\beta_c = 0.99$

**Reinforced steel**
- modulus of elasticity for $\phi 6$ rods made of A-II steel, $\phi 10$ rods made of A-III steel $E_s = 200$ GPa
- yield stress for $\phi 10$ rods made of A-III steel $f_y = 410$ MPa, for $\phi 6$ rods made of A-II steel $f_s = 355$ MPa
- Poisson’s ratio $\nu_s = 0.3$
- steel density $\rho_s = 7800$ kg/m$^3$

**Steel support and steel loading plate**
- modulus of elasticity $E_s = 210$ GPa
- Poisson’s ratio $\nu_s = 0.3$
- steel density $\rho_s = 7800$ kg/m$^3$.

In the second version of the numerical solution, the concrete model is described by the five-parameter failure surface of high strength concrete which is compressed and tensed, in accordance with the William-Warnke theory (Fig. 6) and author’s own proposal of the surface evolution in the function of its deformation (Fig. 5).

William and Warnke [5] presented a more detailed approximation of the concrete failure surface with a five-parameter model. In order to describe the parabolic shape of the failure surface meridians, the tree-parameter model was supplemented with two additional parameters. The criterion of concrete fracture in the complex stress condition is described by the equation:

$$\frac{F}{f_c} - S \geq 0$$

where: $F$ - the function of the principal stress state $\sigma_y, \sigma_x, \sigma_z$ in the direction of the Cartesian coordinate system $x, y, z$, $S$ - failure surface dependant on the principal stresses $\sigma_1, \sigma_2, \sigma_3$, where $\sigma_1 = \max(\sigma_y, \sigma_x, \sigma_z)$, $\sigma_3 = \min(\sigma_y, \sigma_x, \sigma_z)$ and five parameters: $f_c$ - ultimate uniaxial compressive strength (causing crush), $f_t$ - ultimate uniaxial tensile strength (causing failure), $f_{cb}$ - ultimate biaxial compressive strength (causing crush), $f_{1c}$ - ultimate compressive strength for a state of biaxial compression superimposed on hydrostatic stress state $\sigma_h^a$ (causing crush) and $f_2$ - ultimate compressive strength for a state of uniaxial compression superimposed on hydrostatic stress state $\sigma_h^a$.

In the second version of the numerical solution, the concrete failure surface is used as a criterion of destruction according to the following interpretation. The material is destroyed if inequality (7) is fulfilled. The state of failure can be distinguished as the state of cracking, if any principal stress is tensing, or in the state of crushing, if all the principal stresses are compressing.
stress $\sigma^t$. The description of concrete failure is defined in four domains of stresses:
- $0 \geq \sigma_1 \geq \sigma_2 \geq \sigma_3$ (compression - compression - compression)
- $\sigma_1 \geq 0 \geq \sigma_2 \geq \sigma_3$ (tensile - compression - compression)
- $\sigma_1 \geq \sigma_2 \geq 0 \geq \sigma_3$ (tensile - tensile - compression)
- $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0$ (tensile - tensile - tensile).

In each range of strain, independent functions $F_1, F_2, F_3, F_4$ and $S_1, S_2, S_3, S_4$ describe the function state of stresses $F$ and failure surface $S$.

The presented model illustrates the main features of concrete failure and determines the conical surface with the curvilinear slant height and the base constituted from three elliptic segments. The values of the parameters characterising this model are easy to determine with standard strength tests. It contains all three stress constants equal to average stress $\sigma^c, F$ and angle $\theta$. Moreover, it ensures smoothness of the failure surface in a wide range of parameters and describes four other models of failure surface that are used in the problems of concrete destruction in previously determined constant parameters.

Apart from the above presented parameters of constitutive models, the properties of the construction material of the BP-1a beam are defined by additional parameters describing the failure surface of high strength concrete, calculated with the equations presented in the works by William and Warnke [5]:

**Additonal parameters for high strength concrete**

- ultimate biaxial compressive strength, $f_{cb} = 1.2 f_c$, $f_{cb} = 97.4$ MPa
- ultimate compressive strength for state of biaxial compression superimposed on hydrostatic stress state $\sigma^h, f_1 = 1.45 f_s$, $f_1 = 117.7$ MPa
- ultimate compressive strength for state of uniaxial compression superimposed on hydrostatic stress state $\sigma^h, f_2 = 1.725 f_c$, $f_2 = 140.1$ MPa
- ambient hydrostatic stress state describing average distribution of normal stresses $\sigma^h = 1.73 f_c, \sigma^h = 140.5$ MPa
- multiplier for amount of tensile stress relaxation $T_s = 1$.

The material models in the third version of the numerical solutions are identical to the above presented first proposal of the numerical solution. In the first two variants of the solution, Crisfield’s arc-length method was used. Yet, in the third proposition the Newton-Raphson method with adaptive descent was used.

All the numerical calculations were made for an elastic and perfectly plastic model of reinforced steel (Fig. 7) and the elastic and brittle model of concrete, with softening in tension (Fig. 5b).

**ANALYSIS OF FAILURE CONDITION**

Figure 8 presents the maps of crack patterns for various values of load obtained for three proposals of numerical solution for a model BP-1a beam. In the first and third numerical solution, the border surface of compressed and tensed high strength concrete is described by three material parameters, while in the second solution, the border surface is characterised by five parameters. In both solutions in Crisfield’s method, the first cracks are formed in the section of constant moment for destructing load $F_{cr}$, therefore at the same moment as in the experimental value of the destructing load $F_{cr} = 15$ kN. The difference between the calculated and experimental value of the destructing load is c.a. 0.7%. Yet, in the Newton-Raphson method with adaptive descent the first crack is made in the section of typical bending for the load value $F_{cr} = 15.2$ kN, which makes c.a. a 2% difference in comparison to the numerical methods of arc-length.

In the first two numerical solutions, the flexural cracks propagate in the area of constant moment up to the load of 20 kN. Further increase causes propagation of the crack zone horizontally towards the support. In the third solution, the processes of cracking are quicker up to the moment of steel plastification. In all of the presented solutions, a significant increase in cracks can be observed for the load of 25 kN after reinforcing steel plastification. The cracks caused by compression in the load application zone and crushing in the bending zone are clearly noticeable. In both solutions with Crisfield’s method, identical images of smeared cracks were obtained up to the load of 25 kN. In the final stages of loading, from the load of 27 kN up to the destruction moment, enhanced devastation was observed in the bending area that was characterised by new cracks in the surfaces perpendicular to the then non-cracked surfaces, accompanied by a relatively constant number of cracks in the supporting area.
Three-parameter failure surface (Newton-Raphson method with adaptive descent)

- $F = 15.2\, \text{kN}$
- $F = 25\, \text{kN}$
- $F = 13.3\, \text{kN}$
- $F = 27\, \text{kN}$
- $F = 21\, \text{kN}$
- $F = 31.8\, \text{kN}$

Three-parameter and five-parameter failure surface (Crisfield’s arc length method [A-L])

- $F = 14.9\, \text{kN}$
- $F = 25\, \text{kN}$
- $F = 13.3\, \text{kN}$
- $F = 27\, \text{kN}$
- $F = 21\, \text{kN}$
- $F = 31.3\, \text{kN}$

Three-parameter failure surface (Crisfield’s arc length method [A-L])

- $F = 27\, \text{kN}$
- $F = 29.7\, \text{kN}$

Five-parameter failure surface (Crisfield’s arc length method [A-L])

- $F = 27\, \text{kN}$
- $F = 29.7\, \text{kN}$

Fig. 9. Experimental [1] and numerical images of cracking BP-1a beam for load $F = 23\, \text{kN}$
Rys. 9. Eksperymentalny [1] i numeryczne obrazy zarysowania belki BP-1a dla sily $F = 23\, \text{kN}$

SUMMARY

The obtained numerical results supported by experimental results prove the fact that the fragility of high strength concretes in flexural beams is significantly limited by the good relation between the concrete and the reinforcement. The clear correlation of the experimental and numerical results depends on appropriate selection of the linear and nonlinear material properties.

All the obtained numerical results are in qualitative agreement with the experimental results in terms of their location, direction and concentration. In the case of the Newton-Raphson method with adaptive descent, a slightly larger zone of smeared cracks was observed. The numerical images of smeared cracks obtained by Crisfield’s arc length method are best to reflect the real failure surfaces in terms of quantity. However, the images of cracks obtained by the Newton-Raphson method are characterised by better location and concentration. Even better images of cracks in the solutions obtained by Crisfield’s method can be obtained by limiting the minimum step of load increase, which requires a longer time for numerical calculations.

REFERENCES


